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MODAL PARAMETER EXTRACTION OF Z24 BRIDGE DATA

Darby J. Luscher[1], James M. W. Brownjohn[2], Hoon Sohn[3], and Charles R. Farrar[3]

1. Department of Civil Engineering, 205 Cobleigh Hall
Montana State University, Bozeman, MT 59717
2. School of Civil and Structural Engineering, Nanyang Technological University
Blk N1, 50, Nanyang Avenue, Singapore 639798
3. Engineering Sciences & Applications Division, Engineering Analysis Group, M/S C926
Los Alamos National Laboratory, Los Alamos, NM 87545

1. ABSTRACT

The vibration data obtained from ambient, drop-weight, and shaker excitation tests of the Z24 Bridge in Switzerland are analyzed to extract modal parameters such as natural frequencies, damping ratios, and mode shapes. System identification techniques including Frequency Domain Decomposition and Eigensystem Realization Algorithm are employed for the extraction of modal parameters and the stationarity of the bridge's dynamic response is also investigated using time-frequency analysis.

2. INTRODUCTION

A large assortment of system identification techniques is available for systems with measurable inputs and output responses. However, for conventional civil structures, the introduction of external input forces, and the measurement of those inputs become difficult. Thus, there is some desire to find system characterization techniques that can be applied effectively to the measured responses of ambient environmental conditions. Additionally, it would be beneficial if the selected techniques minimize user interaction in order to reduce variability in results.

This paper presents two system identification techniques for extracting modal parameters: Frequency Domain Decomposition (FDD) and Eigensystem Realization Algorithm (ERA). A comparison is made between the results when the methods are applied to separate sets of data from ambient, drop-weight, and shaker excitation testing of the Z24 Bridge in Switzerland. Also, the condition of system stationarity, which is assumed in most modal extraction techniques, is investigated using a time-frequency domain analysis.

3. DESCRIPTION OF THE Z24 BRIDGE TEST

The Z24 is a slightly skewed three-span concrete bridge. Each span consists of a continuous post-tensioned two-box girder supported by concrete piers at each end. The center span is approximately 30 meters in length while each end span is 14 meters.

Vibration testing of the Z24 Bridge consisted of measuring its time-history response to ambient as well as forced input environmental conditions with force-balanced accelerometers. To obtain a high resolution of modal information, a fine mesh density of accelerometers was desired. At the time of testing, an excessively large number of accelerometers were not feasible. To maintain a significant mesh density and still allow adequate recovery of modal parameters, a roving sensor acquisition methodology was adopted. Testing was carried out in two separate series of nine setups. The first series of testing was performed under forced-vibration conditions. At each setup, the roving accelerometers were installed at new locations along the length of the bridge and shakers generated random signals in the frequency range of 3-30 Hz. The response at each sensor was measured. Although the input was measured simultaneously with the response, these data are neglected in the following analyses. After recording time-history response from shaker input, a drop-weight input was introduced to the system. Time-history response from the drop-weight input was recorded separately from the shaker excitation data. After repeating this procedure at each setup, the series of setups was iterated through again to record ambient vibration response of the bridge.

4. INVESTIGATION OF SYSTEM STATIONARITY

System identification techniques, ERA and FDD, presented later rely upon the assumption that the system is stationary with respect to time. First, the validity of this stationarity assumption is investigated by a spectrogram analysis.

Spectrogram computes the time-dependent Fourier transform of a signal using a sliding window. This form of Fourier transform is also known as the short-time Fourier transform. Time histories from the first and last instrumentation setups at a reference DOF is concatenated in series. Then, the spectrogram splits the concatenated signal into overlapping segments and applies time window such as a Hanning window to each segment. Next, the spectrogram procedure computes the discrete-time Fourier transform of each segment to produce an estimate of the short-term frequency content of the signal over the given time period. Note that for a signal from a time invariant system, the frequency content should not change with respect to time axis. For the ambient and shaker excitation data, the segments consist of 1024 points, while the drop-weight data segments contain 256 points.

Figures 1, 2, and 3 show the plotted spectrograms for shaker, ambient, and drop-weight data, respectively. For a linear time-invariant system a spectrogram consists of a horizontal band located at each natural frequency with thickness proportional to the damping associated with that mode. Vertical lines are indicative of energy transfer between modes or non-stationarity of the system with respect to time. Figure 1 shows relatively uniform horizontal bands of energy for the shaker data. There is no obvious discontinuity in the spectrogram at the concatenation point (time = 655 sec).

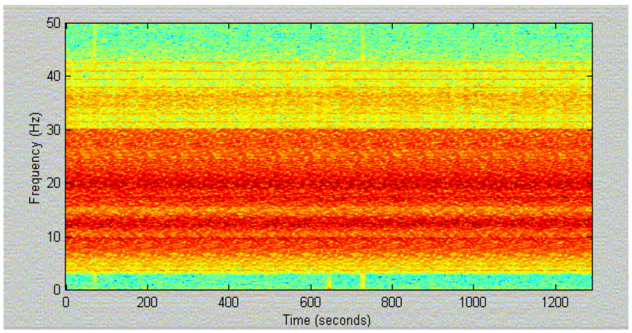


Figure 1: Spectrogram of the shaker test.

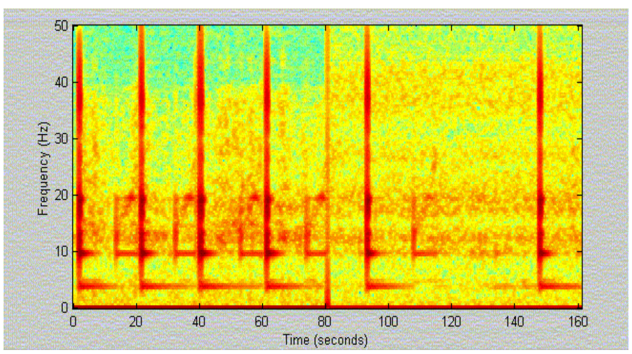


Figure 2: Spectrogram of the drop-weight test.

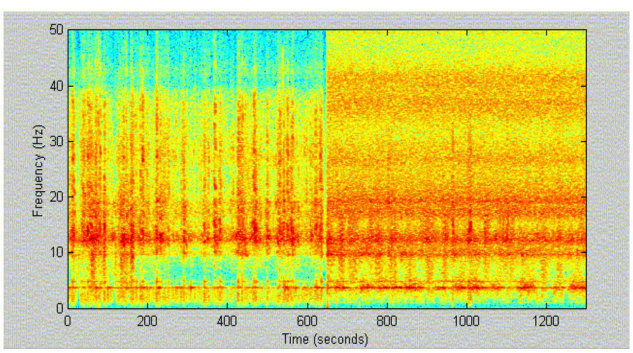


Figure 3: Spectrogram of the ambient test.

On the other hand, the spectrograms of shaker and ambient excitation data show a significant transfer of energy between modes and an obvious discontinuity at the concatenation point (see Figures 2 and 3). The vertical marks on Figure 2 represent the excitation input. Because the excitation is of a much higher energy than the subsequent free decaying motion, the spectrogram magnitude is dominated by the spectral content of the input. It is speculated that variations in normal traffic over the bridge mainly attributes to the nonstationarity of the ambient test.

The observed non-stationarity would affect the subsequent analyses. It is likely that an analysis of a non-stationary system would overestimate the damping while a solution for natural frequency may represent a weighted-average value. To be worse, the phase information of mode shapes would be distorted resulting in unreliable mode shapes.

5. ANALYSIS OF Z24 DATA

For the modal parameter extraction of the Z24 Bridge data, FDD and ERA methods are employed. The detailed descriptions of the FDD and ERA methods can be found in Anderson, 2000 and Juang, 1994.

5.1 Frequency Domain Decomposition (FDD)

The frequency domain decomposition (FDD) technique is based upon the Fourier transform of the time-history into the frequency domain. It is similar to the classical peak-picking method of finding maxima in the power spectral density matrix (PSD) to locate natural frequencies and utilizing the shape of the PSD near natural frequencies to estimate damping ratios.

In the FDD technique applied here, the spectral matrix is formed from the time-history response data by applying a discrete Fourier transform. The spectral matrix is then decomposed by Singular Value Decomposition at each frequency point. Frequency locations of maxima in the first-singular value are estimated as the natural frequencies and the damping estimated using the half-power bandwidth method applied to the singular value versus frequency plot. At identified natural frequencies the mode shapes are estimated from the singular vectors.

Because the data acquisition is conducted nine times by moving measurement points while maintaining a common reference point, additional efforts are required to extract modal parameters. It should be noted that for all of the following analyses any input data was disregarded. Two different approaches were employed in this study: In the first approach, which is called FDD1 hereafter, the FDD technique was performed separately on each set of data. Mode shapes in each setup were normalized with respect to a common reference point. The global mode shapes were then obtained by concatenating the recovered mode shapes from each setup. The averages of natural frequencies and damping ratios from each setup are reported here.

Table 1 shows the averaged natural frequency and damping ratio for each mode as determined from each type excitation. The natural frequencies from different excitations match reasonably well. However, note that the fourth (second-torsion) mode was not recovered from the drop-weight or shaker excitation. This result is attributed to the biased geometry of the inputs. Ambient vibration excited each of the lower modes relatively equally. While the ambient test could not excite the higher modes well enough to recover reasonable mode shapes or damping values, the ambient test enabled recovery of the fourth mode, which was not initially recovered from the shaker or drop-weight data. MAC values between modes recovered from different forcing conditions are presented in Table 2.

Table 1: Modal parameters obtained by FDD1 method

Mode	Shaker		Drop		Ambient	
	f	ζ	*f*	ζ	*f*	ζ
1	3.84	0.93	3.85	1.20	3.85	0.80
2	4.80	2.34	4.73	5.84	4.89	1.52
3	9.69	2.80	9.77	1.92	9.82	1.83
4	N/A	N/A	N/A	N/A	10.30	1.74
5	12.49	5.07	13.04	7.00	12.70	6.05
6	19.59	5.00	18.99	3.66	N/A	N/A
7	26.75	5.13	N/A	N/A	N/A	N/A

Table 2: MAC values for mode shapes obtained from three different excitation types by FDD1 method

Test type	*MAC(i, i)**				
	1	2	3	4	5
Shaker vs. Ambient	0.99	0.97	0.89	N/A	0.00
Shaker vs. Drop	0.99	0.47	0.94	N/A	0.02
Drop vs. Ambient	1.00	0.58	0.97	N/A	0.78

$^*MAC(i,j) = (\phi_i^T \phi_j)^2 \Big/ (\phi_i^T \phi_i)(\phi_j^T \phi_j)$.

Next, a second variation of the FDD method (FDD2) was applied to the same Z24 Bridge data. In this case, data from the nine separate setups were treated as though they were realized simultaneously. That is, the time series from all nine setups were combined together, and the singular value decomposition (SVD) was conducted on the combined data set. FDD was applied only to recover natural modes and damping values. Mode shapes could not be directly recovered using this approach because of the loss of phase information after combining data from multiple setups.

To obtain mode shapes, the SVD was repeated on data from each setup separately at the previously estimated natural frequency values by FDD2. Similar to FDD1, mode shapes from each setup were then normalized to the reference point, and concatenated together. This alternative method shows only marginal improvement. Results for the natural frequencies and damping ratios estimated from FDD1 are presented in Table 3, and the corresponding MAC values are shown in Table 4.

Table 3: Modal parameters obtained by FDD2 method

Mode	Shaker		Drop		Ambient	
	f	ζ	*f*	ζ	*f*	ζ
1	3.85	1.08	3.85	0.96	3.85	1.02
2	4.81	2.19	4.80	2.76	4.89	1.72
3	9.72	2.13	9.73	1.79	9.74	1.43
4	N/A	N/A	N/A	N/A	10.27	1.70
5	12.62	5.60	13.05	5.24	13.25	5.86
6	19.56	4.67	19.19	3.15	20.25	7.40
7	26.65	5.15	N/A	N/A	N/A	N/A

Table 4: MAC values for mode shapes obtained from three different excitation types and FDD2

Test type	*MAC(i, i)**				
	1	2	3	4	5
Shaker vs. Ambient	1.00	0.96	0.86	N/A	0.03
Shaker vs. Drop	0.99	0.45	0.92	N/A	0.02
Drop vs. Ambient	1.00	0.54	0.96	N/A	0.86

A subsequent attempt was made to recover the fourth mode from drop-weight and shaker data. The orthogonality condition of mode shapes was used to find the fourth natural frequency. Because the fourth mode should be, in theory, orthogonal to the third mode, the MAC value between two perfectly orthogonal modes should equal zero. Therefore, the fourth mode could be located by plotting the minimum of *MAC(3,j)* for different frequency values. Also, the natural frequencies were searched such that the MAC values between the mode shapes, which were obtained from different excitation types, are maximized. Frequencies that optimized the correlation between modes of separate data sets and the MAC values corresponding to these operational deflection shapes are listed in Tables 5 and 6, respectively. Because the correlation efforts had little effect on modal parameters recovered from the drop-weight data, the MAC values for modes recovered from those data are omitted.

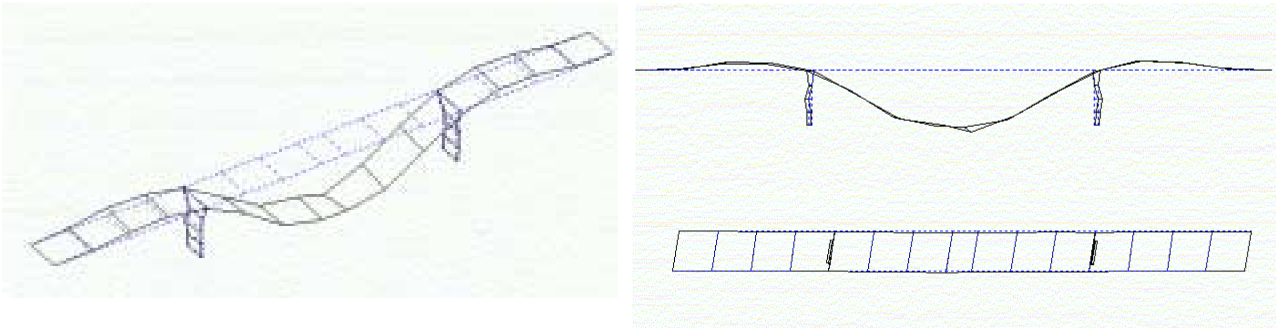
For the drop-weight data, no real improvement was gained in the correlation of mode shapes recovered from the other two data sets. The mode shapes from ambient and shaker-excitation data realized a significant improvement in the correlation between the third, fourth, and fifth mode shapes. The frequency values changed only slightly for a significant change in the correlation of mode shapes. This investigation does not strengthen the confidence in the results from previous analyses, but does lend some insight to the sensitivity of experimental modal analysis upon stationarity and stabilization of natural frequencies. The operating deflection shapes for the frequencies listed in Table 5 obtained from the shaker data are plotted in Figure 1. A reasonable representation of the first seven mode shapes is obtained.

Table 5: Frequencies that maximized the correlation between different excitation types

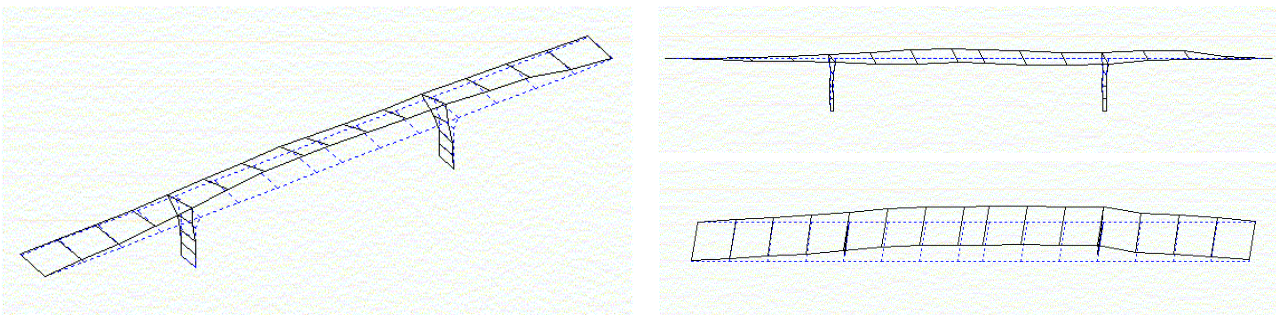
Mode	Shaker	Drop	Ambient
	f	*f*	*f*
1	3.85	3.85	3.85
2	4.81	4.80	4.85
3	9.73	9.73	9.73
4	10.10	N/A	10.23
5	12.61	13.05	12.36
6	19.56	19.19	N/A
7	26.56	N/A	N/A

Table 6: MAC values for the operating deflection shapes obtained at the frequency identified in Table 5

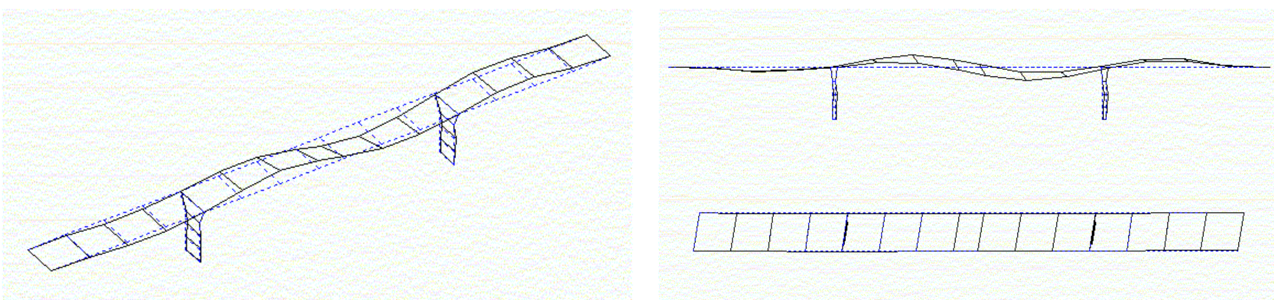
MAC values from Shaker vs. Ambient					
Mode	1	2	3	4	5
1	1.00	0.01	0.00	0.01	0.00
2	0.00	0.98	0.00	0.00	0.00
3	0.01	0.00	0.90	0.02	0.00
4	0.00	0.00	0.12	0.88	0.00
5	0.00	0.00	0.01	0.02	0.97



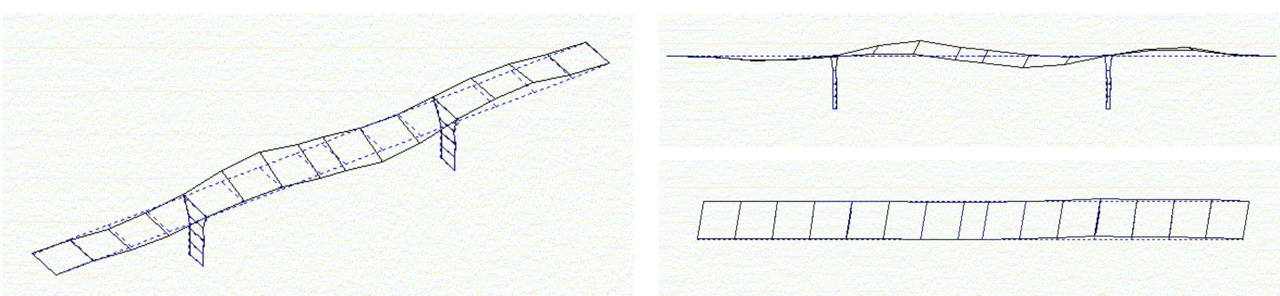
(a) The first mode shape



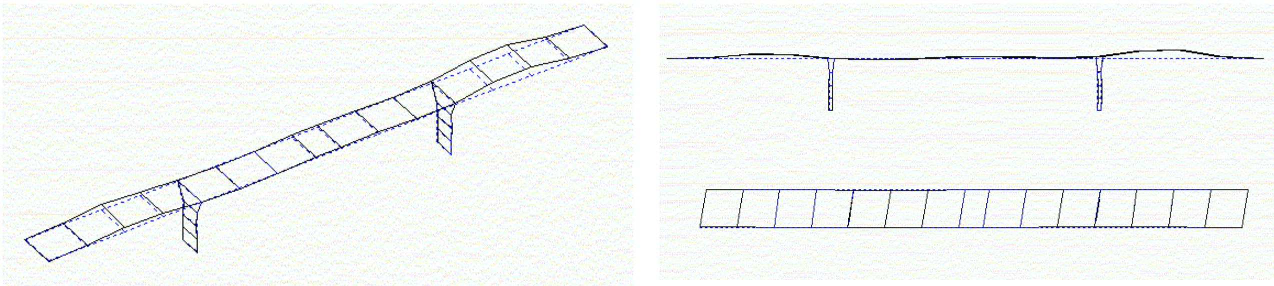
(b) The second mode shape



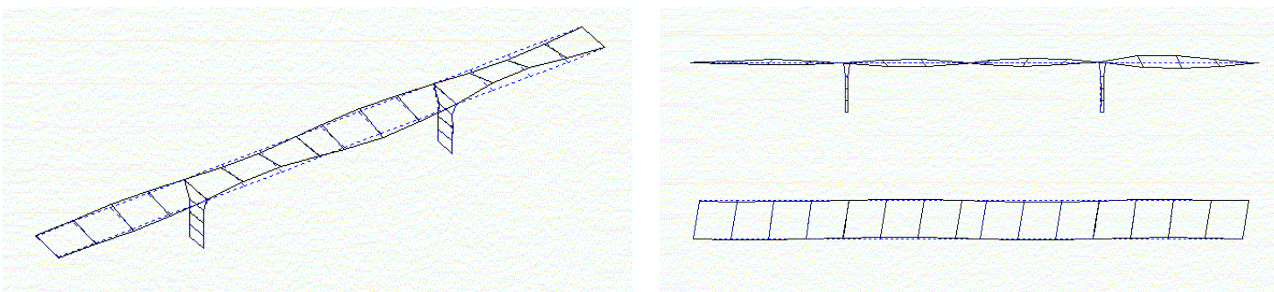
(c) The third mode shape



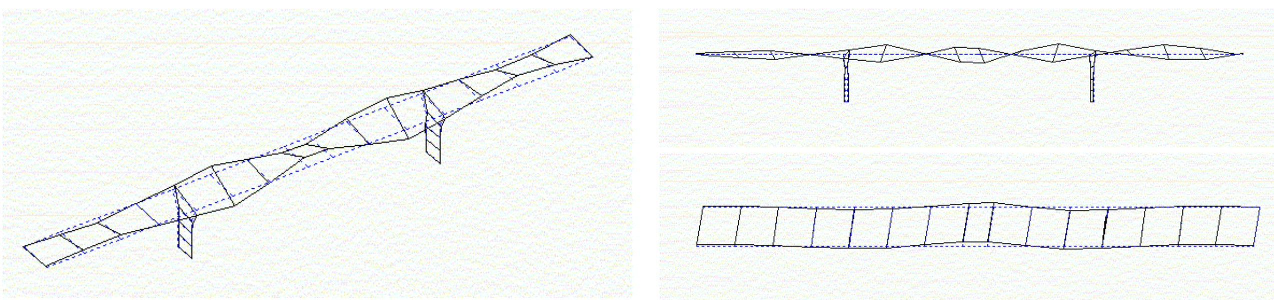
(d) The fourth mode shape



(e) The fifth mode shape



(f) The sixth mode shape



(g) The seventh mode shape

Figure 1: Mode shapes obtained from the shaker excitation data

5.2 Eigensystem Realization Algorithm (ERA)

The Eigensystem Realization Algorithm is an analysis of system parameters based on the relationship of the Hankel matrix of Markov parameters to the state-space matrices that define the system. Markov parameters are determined by inverse Fourier transform of the cross-power spectral matrix estimated from the time-history data. The hankel matrix is constructed from the Markov parameters at k = 1 and k = 0. The hankel matrix is then decomposed and the state-space matrices are estimated. From the hankel- and state-space matrices, the modal characteristics are recovered.

For all the excitation cases analyzed for the Z24 Bridge, the ERA analysis is conducted without the input record. That is, the analysis is performed by ignoring the input data from the shaker tests.

The power-spectral densities of the response data from each setup were mapped together for estimation of the Markov parameters and the subsequent Hankel matrix construction. Note that for the shaker-excitation data, the input channels were disregarded for the analysis.

There are three indicators developed for use with the ERA (Pappa, 1992): Extended Mode Amplitude Coherence (EMAC), Modal Phase Collinearity (MPC), and Consistent Mode Indicator (CMI), which is the product of EMCA and MPC. EMAC is a measure of how accurately a particular mode projects forward onto the impulse response data. MPC is a measure of how collinear the phase of the components of a particular complex mode are. If the phases are perfectly in phase or out of phase with each other, this mode exactly has proportional damping and can be completely represented by the corresponding real mode shape. That is, EMAC is a temporal quality measure and MPC is a spatial quality measure. Modes identified by the ERA method are only retained if the EMAC, MPC, and CMI values greater than 0.7, 0.7, and 0.5 respectively. An attempt is made to visually confirm the mode shapes estimated by the ERA method.

Results for frequency and damping of recovered modes are shown in Table 5. Frequencies and damping values from the ERA analysis have a higher variance among the nine data sets than those from the FDD technique. For ERA, the order of the Hankel matrix, which is a collection of impulse response functions at different time points, significantly affects the results. Storage of the Hankel matrix is a critical issue in the analyses presented here because the assembly of a large Hankel matrix was required. Due to a large number of sensors and a limited amount of computational storage, the order of the Hankel matrix was never large enough to stabilize the results. Therefore, it is speculated that the frequencies and damping values obtained are merely rough estimates of the actual system parameters. Mode shapes are visually similar to each other for the different data types, but MAC values are poor. Therefore, the MAC values for the mode shapes from ERA analysis are not reported in this paper.

Table 5: Frequencies and damping ratios identified by ERA

Mode	Shaker		Drop		Ambient	
	f	ζ	f	ζ	f	ζ
1	3.85	1.07	3.97	1.37	3.86	1.03
2	4.98	8.41	4.87	2.67	4.91	3.21
3	9.82	0.96	9.87	1.73	9.80	1.85
4	10.42	1.69	N/A	N/A	N/A	N/A
5	12.62	3.74	12.69	11.26	12.37	2.37
6	19.64	5.08	19.35	2.61	19.41	2.23

6. CONCLUSIONS

The vibration data obtained from ambient, drop-weight, and shaker excitation tests of the Z24 Bridge in Switzerland were analyzed to extract modal parameters such as natural frequencies, damping ratios, and mode shapes.

The spectrogram analysis reveals the significant nonstationarity for the data obtained from the drop-weight and ambient tests. It is believed that the quality of the identified modal parameters is degraded as a result of this nonstationarity. The forced-vibration analyses using the shaker and drop-weight tests have missed the second torsional mode. It is speculated the placement of the shaker or drop-weight has prevented direct excitation of the second torsional mode. On the other hand, the ambient excitation proved to be effective at exciting lower frequency modes.

The dynamic characteristics of the Z24 Bridge are identified through FDD and ERA techniques. An advantage of the ERA method is that it could be easily adapted to an automated structural characterization method minimizing the modal parameter variability caused by user interaction. The FDD technique more accurately estimates the modal parameters of the Z24 Bridge than the ERA method. However, the poor performance of ERA could be attributed to the limited storage capacity, which prevent the assembly of a large Hankel matrix, not an explicit deficiency in ERA. For either of these techniques, system stationarity should be given a consideration. To place confidence in results an attempt should be made to minimize conditions that result in non-stationarity.

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